Recent shell-model studies of nuclear structure and applications to radiative neutron capture

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IN2P3 theory project: “Photoabsorption Strength Functions from CI method”

NUSPIN, 25.06.2019
R-process nucleosynthesis

Nuclear models are needed to provide input for r-process simulations: masses, level densities ($\rho$), $\beta$ half-lives, $\gamma$-Strength Functions ($f_{XL}$), fission barriers...

Radiative neutron capture: resonant capture

\[ \sigma_{(n,\gamma)}^{\mu \nu}(E_i, n) = \frac{\pi \hbar^2}{2M_{i,n}E_{i,n}} \frac{1}{(2J_i^{\mu} + 1)(2J_n + 1)} \sum_{J, \pi} (2J + 1) \frac{T_n^{\mu} T_\gamma^{\nu}}{T_n^{\mu} + T_\gamma^{\nu}} \]

for \( E_n \sim \text{keV} \)

\[ T_n^{\mu} \gg T_\gamma^{\nu} \longrightarrow \sigma^{\mu \nu} \sim T_\gamma^{\nu} \]

\( E_{i,n}, M_{i,n} \) - center-of-mass energy, reduced mass of the system

\( J_n = 1/2 \)-neutron spin

\( T_n^{\mu} = T_n(E, J, \pi; E_i^{\mu}, J_i^{\mu}, \pi_i^{\mu}) \)

\( T_\gamma^{\nu} = T_\gamma(E, J, \pi; E_m^{\nu}, J_m^{\nu}, \pi_m^{\nu}) \) - transmission coefficients

For a given multipolarity

\[ T_{XL}(E, J, \pi, E^\gamma, J^\gamma, \pi^\gamma) = 2\pi E_{\gamma}^{2L+1} f_{XL}(E, E_{\gamma}) \]

Test, using SM, the key ingredients of Hauser-Feshbach calculations:

- description of \( \gamma \) emission spectra
- Brink-Axel hypothesis
Radiative neutron capture: direct capture

\[ \sigma_{DC}(E) = \sum_{f=0}^{\infty} S_f \sigma_{dis}(E) \]

\[ + \langle S \rangle \int_{E_x}^{S_n} \sum_{J_f, \pi_f} \rho(E_f, J_f, \pi_f) \times \sigma_{cont}^f dE_f \]

If no experimental data available:
- use combinatorial model for the level density with \( \langle S \rangle = 0.5 \)

- The key ingredients: low-energy levels and spectroscopic factors
- Validate theoretical approximations (HFB) in exotic nuclei using SM predictions

Shell model: generalities

Shell model relies on the possibility of diagonalizing the Hamiltonian matrix and deriving (constraining empirically) a suitable effective interaction.

\[ H_{\text{eff}} |\psi_{\text{eff}}\rangle = E |\psi_{\text{eff}}\rangle \]

- **Direct capture:** knowledge of the lowest-lying levels (energies and spectroscopic factors) \( \rightarrow \) quality of the effective Hamiltonian

- **Resonant capture:** knowledge of statistical properties (energies and transitions in nuclear continuum) \( \rightarrow \) possibility of computing of hundreds of nuclear levels
Lenzi-Nowacki-Poves-Sieja interaction

- Based on realistic TBME
- New fit of the pf shell (KB3GR, E. Caurier)
- Monopole corrections

Used successfully in over 50 papers (masses, spectroscopy, transitions, spectroscopic factors...)

$^{60}$Fe spectroscopic factors

S. Giron, PhD Orsay, 2012

$^{76}$Ni from $(p,2p)$ reaction

$^7_{6}Ni$ from $(p,2p)$ reaction

Proton gap $Z = 28$ estimated to be 4.4MeV in $^{76}$Ni and 5.2MeV in $^{78}$Ni, compatible with other recent studies in the region.
Direct capture rates using SM results

$^{64}\text{Ni}(n,\gamma)^{65}\text{Ni}$
Direct capture rates using SM: Ni & Cr

![Graphs showing direct capture rates for Ni and Cr](image)
Direct capture rates using SM results: chromiums

- Great sensitivity to the structure details in very neutron-rich nuclei.

\[ \sigma_{\text{DC}} \text{(mb)} \] vs. \[ E_n \text{ (keV)} \]

\[ 71^{\text{Cr}} \]

- Intrinsic \( \langle S \rangle = 0.5 \)

Excitation energy \( E_{\text{exc}} \) vs. spin factors \( S_f \): intrinsic, SM

Kamila Sieja (IPHC)
Direct capture using SM: triaxial nuclei above $^{78}$Ni

Calculations with NI78-II interaction
Prediction of triaxiality above $^{78}$Ni


Spectroscopy of odd-N nuclei past $N = 50$ of interest for nuclear models
Going beyond usual SM applications: $\gamma$-decay

**photoabsorption (PSF)**
- use Lanczos Strength Function method with a large number of iterations

**EXP:** $\sum B(E1)=0.49 \pm 0.16$ e$^2$fm$^2$ (6-10MeV)
  

**THEO:** $\sum B(E1)=0.485$ e$^2$fm$^2$ (0-10MeV)

**$\gamma$-decay (RSF)**
- calculate desired number of low lying states using standard SM diagonalization techniques
- obtain the averages and radiative strength functions from relations:

  \[
  f_{M1/E1}(E_\gamma) = 16\pi/9(\hbar c)^3 S_{M1/E1}(E_\gamma) \\
  S_{M1/E1} = \langle B(M1/E1) \rangle \rho_i(E_i)
  \]

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**Graphs**

- Plot showing $B(E1)$ vs. Energy (MeV) with a peak at around 20 MeV.
- Plot showing $f_{E1}$ vs. $E_\gamma$ (MeV) with a sharp peak at 10 MeV, labeled $^{26}$Ne.
Description of the resonance and radiative decay in the same theoretical framework.

Reasonable agreement between QRPA and SM PSF up to 15MeV.

Microscopic SM strength has a non-zero limit for $E_\gamma = 0$. Consistent with the EGLO model.
Electric and Magnetic Dipole Strength at Low Energy

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- all states below $S_n \sim 10$MeV
- 86642 $M1$ matrix elements
- 65670 $E1$ matrix elements

- Overall good agreement with data:
  - the upbend is due to $M1$ transitions
  - the $E1$ pattern is flat with the non-zero $E_\gamma \rightarrow 0$ limit
M1 upbend: general trends


The strength at $E_\gamma = 0$ peaks around shell closures and is flat in deformed nuclei.
Shell structure in the nuclear quasi-continuum

Ratio of $B(M1)$ strength: $\frac{\langle B(M1) \rangle_{(0-2\text{MeV})}}{\langle B(M1) \rangle_{(2-6\text{MeV})}}$

mid-mass nuclei:

sd-shell nuclei:

The ratio peaks towards the edges of the model spaces at $N = 8, 20, 40, 50$.

Some extra shell effects are present in the Ni chain.

In sd-pf nuclei the ratio peaks at $N = 28$ for Ca only.

sd – pf nuclei:

The puzzling case of $^{46}$Ar

- Signs of shell closure in $^{46}$Ar from $2^+$ energies and two-neutron separation energies
- Confusing evidence from $B(E2)$ values
- No shell closure from the $B(M1)$ decay strength!

M. Mougeot, PhD thesis, Orsay 2018
To describe radiative decay, phenomenological low-energy corrections fitted to reproduce SM trends and data are added to microscopic QRPA-Gogny $M1$ and $E1$ PSF:

\[
f_{E1}(\gamma) = f^{QRPA}_{E1}(\gamma) + f_0 U/[1 + e^{(\gamma - \gamma_0)}] \quad (1)
\]

\[
f_{M1}(\gamma) = f^{QRPA}_{M1}(\gamma) + C e^{-\eta \gamma} \quad (2)
\]

- **Upper limit ($0^{lim+}$)**
  
  \[f_0 = 5 \cdot 10^{-10} \text{MeV}^{-4}, \ \gamma_0 = 5 \text{MeV}, \ C = 3 \cdot 10^{-8} \text{MeV}^{-3}, \ \eta = 0.8 \text{MeV}^{-1}\]

- **Lower limit ($0^{lim-}$)**
  
  \[f_0 = 10^{-10} \text{MeV}^{-4}, \ \gamma_0 = 3 \text{MeV}, \ C = 10^{-8} \text{MeV}^{-3}, \ \eta = 0.8 \text{MeV}^{-1}\]
Impact on radiative neutron capture

\[ \langle \Gamma \gamma \rangle \langle \sigma \rangle \]

<table>
<thead>
<tr>
<th></th>
<th>\langle \Gamma \gamma \rangle</th>
<th>\langle \sigma \rangle</th>
</tr>
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<tbody>
<tr>
<td>0\text{lim}^- \text{(Comb)}</td>
<td>0.88</td>
<td>1.07</td>
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<tr>
<td>0\text{lim}^- \text{(CT)}</td>
<td>0.74</td>
<td>0.95</td>
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<tr>
<td>0\text{lim}^+ \text{(Comb)}</td>
<td>1.02</td>
<td>1.30</td>
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<tr>
<td>0\text{lim}^+ \text{(CT)}</td>
<td>0.90</td>
<td>1.15</td>
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<tr>
<td>GLO\text{(Comb)}</td>
<td>0.48</td>
<td>0.61</td>
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<tr>
<td>GLO\text{(CT)}</td>
<td>0.38</td>
<td>0.53</td>
</tr>
</tbody>
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\[ ^{135}\text{Ba}(n,\gamma)^{136}\text{Ba} \]

\[ \sigma [\text{mb}] \]

\[ E_n [\text{MeV}] \]

- Voss et al. (1994)
- Musgrove et al. (1974)
- D1M+QRPA
- D1M+QRPA+E1 0\text{lim}^+
- D1M+QRPA+M1 0\text{lim}^+
- D1M+QRPA+0\text{lim}^+
Impact on the radiative capture

MACS ratio at $T = 10^9$ K

- Non-zero limit of the $E1$ strength from SM has small impact on neutron capture: 20 – 50%
- $M1$ upbend can alternate the cross-section by a factor $>10$ in exotic nuclei

S. Goriely, S. Hilaire, S. Péru and K. Sieja, PRC98 (2018) 014327
SM can provide reliable spectroscopic factors and help testing usual theoretical assumptions in cases no experimental data is known → work in progress.

Spectroscopy of neutron-rich nuclei around $^{78}$Ni is still of interest for nuclear models.

$E1/M1$ RSF and PSF can be microscopically obtained within the SM.

Shell effects survive at higher excitation energies and are visible in $M1$ dipole strength functions.

$M1$ upbend has a significant impact on neutron capture cross sections in exotic nuclei: $\times 10$. 